

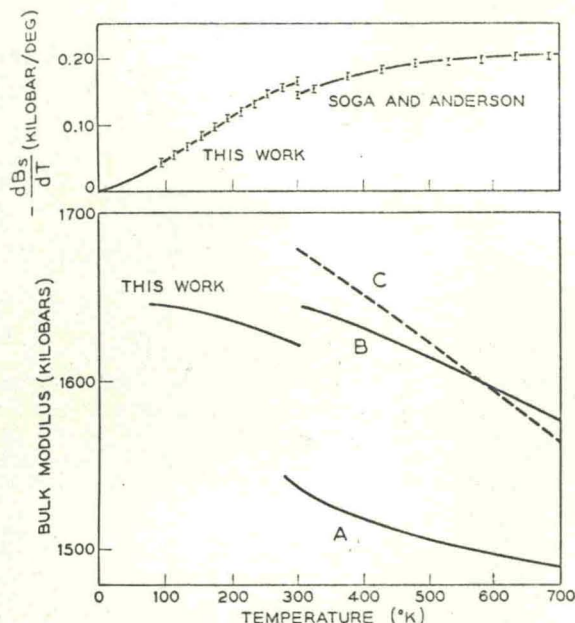


Fig. 2. (Lower) Adiabatic bulk modulus vs. temperature. Lines A and C represent bulk moduli computed from single-crystal data from Refs. 10 and 9, respectively; line B represents polycrystalline data from Ref. 19. (Upper) Value of $-dB_s/dT$ computed from the indicated data. This curve resembles the C_p curve for MgO.

where the Debye function is expressed in deg^{-1} . Equation (3) is plotted as a dashed line in Fig. 3, together with the measured values of B_s . Within a fraction of a percent Eq. (3), which has no disposable constants, agrees with the measured values of B_s .

V. Measurements as a Function of Pressure

(1) By Method of Computing Length Changes

The pressure experiments were made in the equipment described previously^{2,3} and the same corrections were made. The experimental plots of repetition rate frequency with pressure were straight lines within the error of the experiment. Table VI shows the intercepts and the slopes of f_R vs. P for the five velocities (or modes) taken at 23° and -195.8°C. The last column in the table gives the vital information. To compute the elastic constants, information on change of length with pressure and temperature is required. Following the techniques of Cook,¹³ the change in length was computed in terms of the measured frequency ratios associated with v_1 and v_5 .

Table VI. Best-Fit Straight Lines for Variations of f_R with Pressure

Mode	Slope (cps/psi)	Slope (cps/kbar)	Intercept (cps)	Correction to f_R at 30,000 psi or 2.0681 kbars	$\Delta f/f\Delta P$ /kbar
$T = 23^\circ\text{C}$					
1	0.017801	258.18	176,391 ($N = 2$)	-1.5	0.00146370
2	.002399	34.795	127,888 ($N = 2$)	+1.7	.00027203
3	.011105	161.066	196,025 ($N = 2$)	-1.5	.00082166
4	.003637	52.75105	261,033 ($N = 1$)	+6.5	.00040418
5	.012202	176.977	104,917 ($N = 2$)	+0.8	.00168682
$T = -195.8^\circ\text{C}$					
1	0.017848	258.87	179,095 ($N = 2$)	-3.0	0.00144544
2	.002853	41.380	128,496 ($N = 2$)	+2.2	.00032204
3	.010623	154.076	197,504 ($N = 2$)	-1.7	.00078012
4	.004794	69.532	262,223 ($N = 1$)	+6.5	.00053034
5	.011604	168.304	107,637 ($N = 2$)	+4.5	.00156362

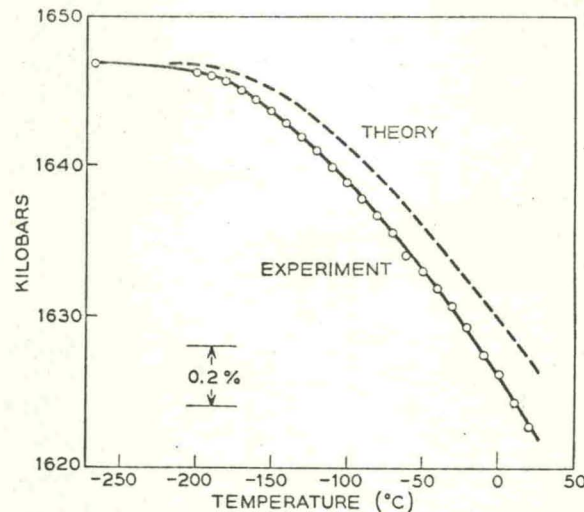


Fig. 3. Comparison of the theoretical curve for bulk modulus vs. temperature, Eq. (3), with experimental data.

The elastic constants at the various pressures and temperatures are reported in Table VII together with the pressure derivatives. The main result is that $dB_s/dP = 4.50$ at 23°C and 4.74 at -195.8°C. The variation of dB_s/dP from very low temperature to room temperature is, therefore, quite small.

(2) Thurston and Brugger's Method of Natural Velocities

Thurston and Brugger¹⁴ related the initial pressure derivatives of the various quantities directly to the initial slope of the curves of frequency vs. stress.

Following this approach, Thurston gave the following explicit formula for the case of cubic symmetry¹⁵:

$$-\frac{dB_s}{dP} = 2c_{11} \left(\frac{f'_1}{f_{10}} \right) - \frac{4}{3} (c_{11} - c_{12}) \left(\frac{f'_5}{f_{50}} \right) + \frac{1}{3} (1 + \alpha \gamma T) \quad (4)$$

where the f' represents the pressure derivatives of the frequency, α is the coefficient of thermal expansion, and γ is the Grüneisen constant; the values of f'_5/f_{50} and f'_1/f_{10} are found in the last column of Table VI. Using these values:

$$\frac{dB_s}{dP} = 4.49 \text{ at } 23^\circ\text{C}$$

$$\frac{dB_s}{dP} = 4.73 \text{ at } -195.8^\circ\text{C}$$

The agreement between these values and the results computed using the method of Cook is satisfactory, consider-